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Microarticle

# A hybrid Scatter/Transform cloaking model

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ABSTRACT

A new Scatter/Transform cloak is developed that combines the light bending of refraction characteristic of a Transform cloak with the scatter cancellation characteristic of a Scatter cloak. The hybrid cloak incorporates both Transform's variable index of refraction with modified linear intrusions to maximize the Scatter cloak effect. Scatter/Transform improved the scattering cross-section of cloaking in a 2-dimensional space to 51.7% compared to only 39.6% or 45.1% respectively with either Scatter or Transform alone. Metamaterials developed with characteristics based on the new ST hybrid cloak will exhibit superior cloaking capabilities.



To cloak an object from an incident wave, the wave is reassembled to propagate onward as if the object was not there. Invisibility cloaks are limited by their mathematical representations. Two widely investigated, fundamentally different mathematical cloaking representations are Transform cloaking and "Scatter", or scattering cancellation cloaking, each with advantages and limitations. Here, a hybrid Scatter/Transform or "ST" cloak is developed which combines the advantages of each.

Invisibility cloaks have a wide range of applications from shielding to preventing the external environment from being disturbed [1–5]. Shielding may be more effective than absorption or reflection of energy, by inducing gradual, perturbations using material parameters that weakly interact with the incident wave. Hence, cloaking materials will tend to exhibit less wear due to heat, vibrations, or other energy emissions; cloaks are also designed to protect or shield the environment from electronic, hydrodynamic, seismic, acoustic or mechanical waves, [1–6]. Mathematical cloaking representations can be generalized to cloak any wave type, in as much as materials can be determined that affect the wave appropriately; in addition to classical models research has also been reported on cloaking matter's quantum mechanical waves [3,4].

Two principal mathematical models of cloaking are Transform [7,8] and Scatter [6,9], which take two different approaches to generate the invisibility cloak. Transform warps a wave around an object by determining the necessary geometric transformation to bend all incoming vectors around an object in the cloaking space. This is accomplished by translating the space surrounding an object such that grid lines bend around the object, but do not touch the object, and incoming vectors return onto an object-free path a certain distance from the object (boundary of the cloak). The distorted coordinate lines comprise a spatial compression that can be built by varying such physical parameters as the refractive index. Scatter, however, focuses on canceling out the interference interactions near the object, to behave as if the object was not there (negates the effects of wave scattering).

Transform cloaking can in principle function for any size or shaped object; however, the transform equations can lead to cloaks with continuously changing or unrealistic (*i.e.* infinite) values of parameters such as the refractive index. The scale at which a Transform cloak is built can be overly sensitive to the size of the incident wavelength, making it a challenge to construct an effective broadband (transform) cloak. Scatter cloaking is much less sensitive to such scale variations to provide a facile broadband cloak, but it is computationally and physically more challenging to design for complex object shapes, and Transform is more effective for a variety of object geometries. To overcome the limitations of either method and develop on overall better or for objects at or larger than the size of the incident wavelength, cloak, here a new synergistic, practical cloaking model is developed.

The top section of Fig. 1 compares the action of Transform and Scatter cloaking. As seen on the right, the Transform cloaking space has been divided into concentric shells. Increasing the index of refraction in near lying shells bends light more sharply when it is closer to the object to be cloaked. Refraction is greater when light is directed toward the object, with the net effect that a wave is transformed to bend around the cloaking region. As shown in the top left of the figure, classical Scatter cloaking utilizes circular intrusions to scatter a wave in proximity to the object (the density of intrusion packing has been reduced for clarity). The majority of

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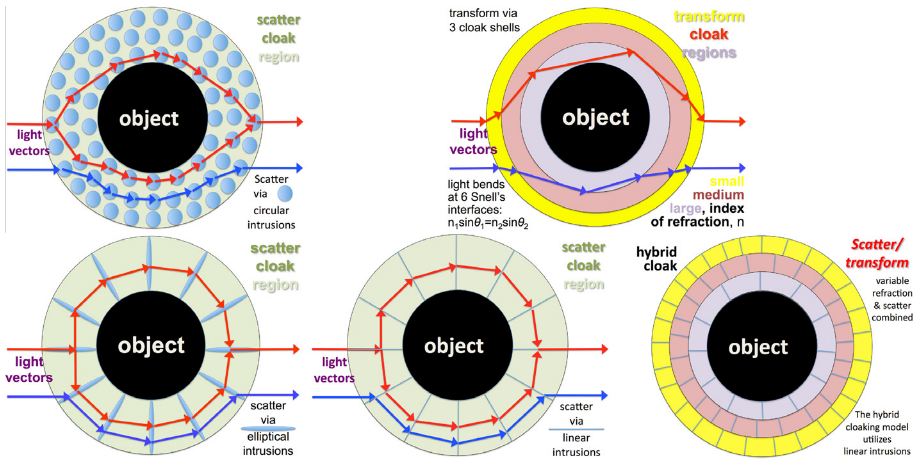


**Fig. 1.** Illustration of (top right) Transform cloaking; (top left) classical Scattering cloaking; (bottom left) Scatter cloaking with ellipsoid intrusions; (bottom middle) modified Scatter cloaking with linear intrusions; (bottom right) ST hybrid cloaking as presented in this study with linear intrusions. The number, size, density, and position of the intrusions are optimized to maximize cloaking. (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this article.)

the light (wave) is scattered in the correct (cloaked) directionality, and other scattered light tends to cancel out. The propagating vectors of the wave are indicated, as other directionalities tend to cancel. In the lower left panel of the figure, ellipsoids improve upon the circular intrusions as more of the wave is directed along the vector components when it is in a direction normal to the shell, and less when parallel to the minor ellipsoid axis [10]. In the bottom middle of Fig. 1, a further modification to bend even more of the wave in the correct direction, using linear, rather than elliptical, intrusions is introduced to provide the highest degree of energy directed tangential to the shell, and eliminate scattering toward the object.

The new ST cloak, as illustrated in the bottom right of Fig. 1, incorporates both Transform's variable index of refraction (Eq. (1)) with intrusions to maximize the Scatter cloak effect. To ensure that the two modes are compatible, intrusions are designed to bend light incrementally (Eq. (2)). The weighted average of the intrusions and matrix for each shell is set to the Transform's requisite refractive index (Eq. (3)) and the difference between intrusions and matrix is set to bend light as in Scatter cloaking (Eq. (4)). The illustrated model utilizes 3 rings of linear intrusions, while the actual mathematical construct examined incorporates 10 concentric shells of increasing refractive index with densely packed linear intrusions. A wave's interactions with the cloak's concentric shells in a 2-D space are calculated by MEEP (the MIT Electromagnetic Equation Propagation software). The cloak combines the light bending of refraction characteristic of a Transform cloak with the scatter cancellation characteristic of a Scatter cloak. The hybrid is constructed utilizing the time iterated Transform and Scatter equations:

$$\text{Transform equation}: \; A_x = R^2/(r_x^2 - r_1^2) \tag{1}$$

$$\text{Linear intrusion scatter-cancellation eq}: \; T_x = r_x/(r_x - r_1) \tag{2}$$

$$\text{Simultaneous co-equations}: \; (L_x + NM_x)/(N+1) = A_x \tag{3}$$

$$T_x = L_x/M_x \tag{4}$$

where $R$ = radius of cloak's outer boundary.

$r_1$ = radius of object; $r_x$ = radius at x from center of object.
$A_x$ = average refractive index at that point.
$N$ = ratio of space taken up by linear to cloak's matrix at inner boundary of that layer.
$T_x$ = relative refractive index between intrusions and matrix in shell.
$L_x$ = refractive index of line in that cloak's shell.
$M_x$ = refractive index of matrix in that cloak's shell.

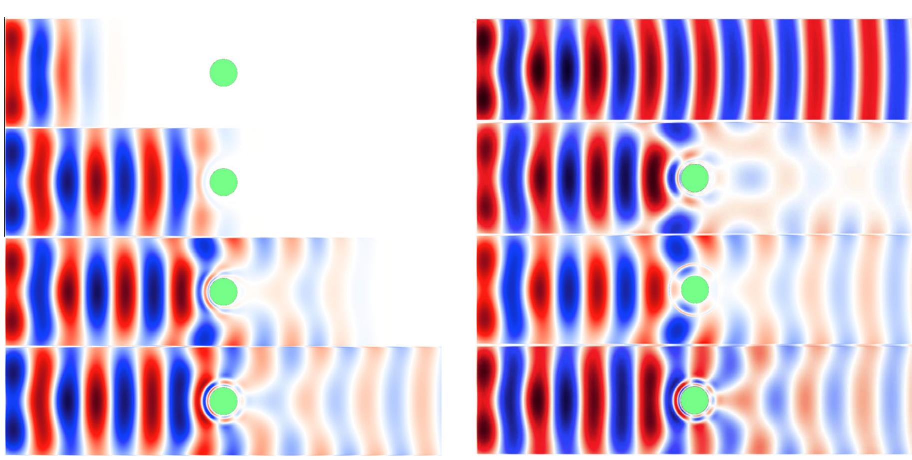

**Fig. 2.** Right side: the new ST cloak in operation (bottom) obscures the object from the incident wave (top) better than the Transform cloak (2nd from top) or Scatter cloak (3rd from top). Each map on the right side is at time, 1000. Left side: the ST cloak in operation (top to bottom) at times, 200, 400, 600, 800.

The efficacy of the new ST model is evident in Fig. 2, which presents the propagation of incident radiation with and without ST cloaking. As evident on the right side of the figure new ST cloak in operation (bottom) obscures the object from the incident wave (top) better than the Transform cloak or Scatter cloak. The Transform cloak outperforms the Scatter cloak (darker red regions) in the space in front of the object, while the Scatter is seen to outperform Transform in the region behind the object. The ST cloak is seen to outperform both in either region. In the cloak calculation, wavelength and time are in relative units to be equally applicable to a wide range of electromagnetic radiation, or shock or acoustic waves although represented with electromagnetic magnitude $E_z$, $H_x$ and $H_y$. In each calculation a wave with wavelength = 40 is incident from the left side in a 320 by 80 cell 2D space, each cell has $17 \times 17$ pixel resolution, and $N = 20$. The object shown in green is perfectly specularly reflective and has radius, $r_1 = 20$. Shown is the $E_z$ component of the wave, not shown are the simultaneous $H_x$ and $H_y$ components. Electrical energy varies from a relative maximum positive value shown in dark red to a relative maximum negative shown in dark blue. The cloaking region consists of 10 concentric circles with radii increasing from 12 to an outer boundary of $R = 20$, and with respective indices of refraction in the Transform and ST cloaks ranging from 39.0 (near the object) to 1.43; the Scatter and ST cloaks are constructed with linear intrusions (as per Fig. 1).

The differential scattering cross-section, SCS, quantifies the degree of cloaking by comparing the total energy scattered by an object normalized to the incident energy density. Differential SCS is measured as the normalized sum of the squares of the relative difference of $E_z$, $H_x$ and $H_y$ in a circular path external to the cloak. Values range from a perfect-cloak value of 100% (all energy transmitted through the object) to a no-cloak value of 0% (all energy reflected by the object). In the Fig. 2 cloak, ST increased SCC to 51.7% compared to 0% without the cloak, whereas Scatter or Transform respectively only increased SCS to 39.6% or 45.1%.

Further refinements of this ST cloak are being pursued to increase the number of cloak shells and to separately vary the permittivity and permeability components of the index of refraction in the cloaked space to further increase the cloak's efficacy. The ST cloaking model is compatible with asymmetric objects, which will be demonstrated in future study. Metamaterials developed with characteristics based on the new ST hybrid cloak presented here will exhibit superior cloaking capabilities.

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